



PERGAMON

International Journal of Solids and Structures 38 (2001) 2069–2081

INTERNATIONAL JOURNAL OF
SOLIDS and
STRUCTURES

www.elsevier.com/locate/ijsolstr

Influence of orifice distribution on the thermal and static properties of hybridly lubricated bearings

Ilmar Ferreira Santos ^{*}, Rodrigo Nicoletti

*Faculdade de Engenharia Mecânica, Dept. de Projeto Mecânico, (FEM/DPM), UNICAMP, Universidade Estadual de Campinas,
13083-970 Campinas, SP, Brazil*

Received 21 July 1999; in revised form 10 January 2000

Abstract

Tilting-pad journal bearings with multiple orifices hybrid lubrication are analysed by means of a thermohydrodynamic theory. Adiabatic boundary conditions are adopted, and a bidimensional model is used to represent the fluid flow behavior in the bearing gap. The influence of operational conditions on the temperature distribution, when different bore arrangements in the pads are used, is discussed and compared to the theoretical values coming from the conventional hydrodynamic case (without radial oil injection). © 2001 Elsevier Science Ltd. All rights reserved.

Keywords: Tilting-pad hybrid bearing; Thermohydrodynamics; Active bearings

1. Introduction

It is well known that tilting-pad journal bearings (TPJBs) are those with the best properties related to dynamic stability. However, such characteristics depend strongly on the rotor velocity. The oil film damping properties decrease as a function of the increase of the rotor angular velocity, leading the rotor-bearing system to instabilities and self-excited vibrations (Flack and Zuck, 1988; Santos and Nicoletti, 1996). Considering these facts, the application of control techniques for improving the rotor stability became a new trend in journal bearing design (Ulbrich and Althaus, 1989; Santos, 1993). Among the control techniques proposed, there is the possibility of injecting oil directly into the bearing gap. Using oil injection through multiple bores machined in the pads, one can obtain an effective increasing of the bearing load capacity (Santos and Russo, 1998), although thermal effects have still not been thoroughly investigated (Santos and Nicoletti, 1998).

For predicting the thermal effects in journal bearings, one has to solve the energy equation simultaneously with the Reynolds equation and the lubricant properties. Since the publication of the generalized Reynolds equation by Dowson (1962), many publications were presented applying thermohydrodynamic theory (THD) to TPJBs (Jones and Martin, 1979; Ettles, 1980; Heshmat and Pinkus, 1986; Tanigushi et al.,

^{*}Corresponding author. Fax: +55-19-289-3722.

E-mail addresses: ilmar@fem.unicamp.br, ifs@iks.dtu.dk (I.F. Santos).

Nomenclature

c	pre-load factor ($c = 1 - h_0/(R_s - R)$)
C_p	oil specific heat
d	distance to the pad center
d_0	orifice diameter
$\mathcal{F}(\bar{y}, \bar{z})$	orifice positioning function
$\mathcal{G}(\bar{y}, \bar{z})$	orifice positioning function
$h(\bar{y})$	bearing gap
h_0	bearing assembled clearance
k_c	oil thermal conductivity
l_0	orifice length
L_y	pad length
L_z	pad width
n	number of pads
$p(\bar{y}, \bar{z})$	oil pressure
P_{inj}	injection oil pressure
Q_r	oil supply flow
R	rotor radius
R_s	pad inner radius
t	time
$T(\bar{y}, \bar{z})$	oil temperature
\bar{T}	average oil temperature over pad
T_{inj}	injection oil temperature
U	rotor surface velocity
V_{inj}	injection velocity
$\bar{x}, \bar{y}, \bar{z}$	auxiliar reference frame
y_o, z_o	coordinates of orifice center
Y_R	vertical rotor position in the inertial reference frame
ε	eccentricity ratio ($\varepsilon = Y_R/h_0$)
ρ	oil density
Ω	rotating speed

1990; Brockwell and Dmochowski, 1992; Fillon and Khonsari, 1996). However, all investigations were done taking into account conventional lubrication mechanisms. The main contribution of this theoretical work is the application of a THD theory to investigate the thermal effects in the oil flow, when different bore arrangements in the pads are used in the hybrid TPJB.

The TPJB lubricated through multiple bores is composed of four tilting shoes (Fig. 1). Each shoe is built of two complementary parts, attached to each other by screws, forming a reservoir, which is filled with lubricant and connected to servovalves. Bores regularly displaced on the surface of the shoes are machined, allowing the fluid flow between the reservoir and bearing gap. The operational principle of such a lubrication is explained in detail in the work of Santos and Russo (1998). A test rig has already been built, and experimental results will be presented in future works (Santos and Scalabrin, 2000). It is important to point out that the bearing is simultaneously supplied by means of a conventional hydrodynamic lubrication between pads, and the goal of this work is the THD investigation of steady state condition in the oil film, when the injection pressures remain time independent.

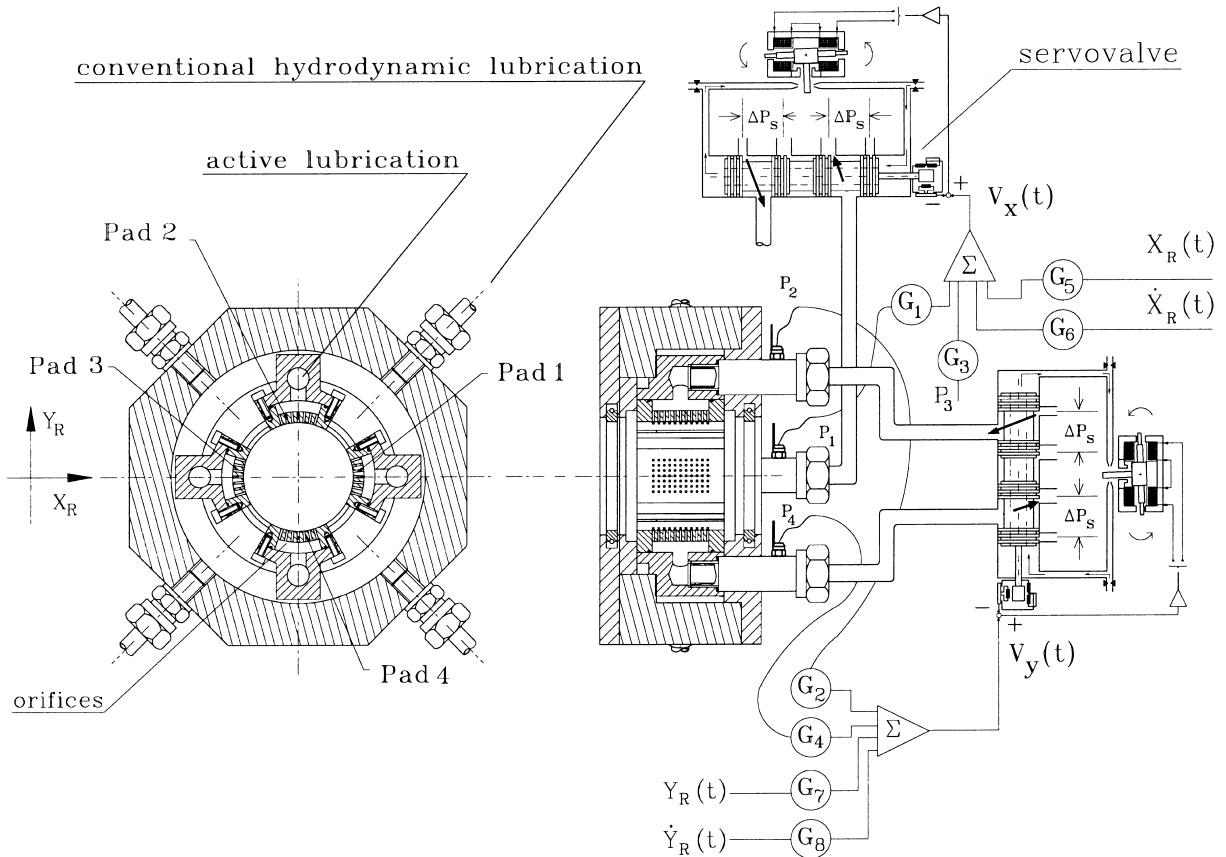


Fig. 1. TPJB with conventional and hybrid lubrication through multiple orifices in the pads (Santos and Russo, 1998).

2. Mathematical modeling

2.1. Modified Reynolds equation considering oil viscosity variations

Adopting an auxiliar reference frame $\bar{x}\bar{y}\bar{z}$ fixed on the pad, where the $\bar{y}\bar{z}$ plane is the pad's surface, one can write the Navier–Stokes equations for a differential volume of fluid in the bearing gap. One assumes that the lubricant is a Newtonian fluid, the flow is laminar and incompressible, the bearing gap is narrow enough to neglect the inertia and body forces terms are considered negligible when compared to the pressure and viscous-effect terms. Considering that the oil viscosity is independent on the radial direction (bidimensional model: $\mu = \mu(\bar{y}, \bar{z})$), one can integrate the Navier–Stokes equations and find the equations of the flow velocity profiles in three directions $\bar{x}\bar{y}\bar{z}$. Inserting these profiles in the continuity equation, where one considers a constant oil density ($\rho = \text{const.}$), and integrating it in the interval $[-h, 0]$ (bearing gap), one arrives to the modified Reynolds equation considering oil viscosity variations, given by

$$\frac{h^3}{\mu} \frac{\partial^2 p}{\partial \bar{y}^2} + \frac{h^3}{\mu} \frac{\partial^2 p}{\partial \bar{z}^2} + \left(\frac{3h^2}{\mu} \frac{\partial h}{\partial \bar{y}} - \frac{h^3}{\mu^2} \frac{\partial \mu}{\partial \bar{y}} \right) \frac{\partial p}{\partial \bar{y}} + \left(\frac{3h^2}{\mu} \frac{\partial h}{\partial \bar{z}} - \frac{h^3}{\mu^2} \frac{\partial \mu}{\partial \bar{z}} \right) \frac{\partial p}{\partial \bar{z}} = 6U \frac{\partial h}{\partial \bar{y}} - 12 \frac{\partial h}{\partial t} + 12V_{\text{inj}}, \quad (1)$$

where p is the pressure and U , the velocity of rotor surface.

Classical boundary conditions are adopted to solve Reynolds equation. Along the whole pad's edge, ambient pressure is considered, as well in the cavitation zones. The injection velocity profile (V_{inj}) is determined considering a completely developed laminar flow inside the orifice. Hence, one gets

$$V_{\text{inj}}(\bar{y}, \bar{z}) = -\frac{1}{4\mu l_0} (P_{\text{inj}} - p) \mathcal{F}(\bar{y}, \bar{z}), \quad (2)$$

where l_0 is the bore length; P_{inj} , the injection pressure, and

$$\mathcal{F}(\bar{y}, \bar{z}) = \begin{cases} \frac{d_0^2}{4} - (\bar{y} - y_o)^2 - (\bar{z} - z_o)^2 & \text{if } (\bar{y} - y_o)^2 + (\bar{z} - z_o)^2 < \frac{d_0^2}{4}, \\ 0 & \text{if } (\bar{y} - y_o)^2 + (\bar{z} - z_o)^2 \geq \frac{d_0^2}{4}, \end{cases} \quad (3)$$

where d_0 is the bore diameter and (y_o, z_o) are the bore coordinates over the pad in the auxiliar reference frame. The function $\mathcal{F}(\bar{y}, \bar{z})$ is responsible for representing the position of the orifices over the pad, and also describes the parabolic shape of the injection flow.

From the modified Reynolds equation (1) and the adopted boundary conditions (2) and (3), one can determine the oil pressure distribution over each pad when setting the rotor-bearing operational conditions.

2.2. The energy equation

Applying the energy conservation principle to a differential volume of fluid in the bearing gap, and adopting the same assumptions to find the Reynolds equation, one achieves the energy equation. Inserting the velocity profiles, calculated from the Navier–stokes equations, and integrating it in the interval $[-h, 0]$, one obtains the energy equation applied to the problem as a function of the bearing gap (h):

$$\begin{aligned} & -\rho C_p h \frac{\partial T}{\partial t} + k_c h \frac{\partial^2 T}{\partial \bar{y}^2} + k_c h \frac{\partial^2 T}{\partial \bar{z}^2} + k_c \frac{\partial T}{\partial \bar{x}} \Big|_0 + \left(\frac{\rho C_p h^3}{12\mu} \frac{\partial p}{\partial \bar{y}} - \frac{\rho C_p U h}{2} \right) \frac{\partial T}{\partial \bar{y}} + \frac{\rho C_p h^3}{12\mu} \frac{\partial p}{\partial \bar{z}} \frac{\partial T}{\partial \bar{z}} \\ & + \rho C_p \left(V_{\text{inj}} - \frac{\partial h}{\partial t} \right) (T - T_{\text{inj}}) = p \left(V_{\text{inj}} - \frac{\partial h}{\partial t} \right) - \frac{4}{3} \frac{\mu}{h} \left(V_{\text{inj}} - \frac{\partial h}{\partial t} \right)^2 - U^2 \frac{\mu}{h} - \frac{h^3}{12\mu} \left[\left(\frac{\partial p}{\partial \bar{y}} \right)^2 + \left(\frac{\partial p}{\partial \bar{z}} \right)^2 \right], \end{aligned} \quad (4)$$

where C_p is the oil specific heat, k_c is the oil thermal conductivity, T_{inj} is the injection oil temperature, and one considers the lubricant viscosity independent on the radial direction, or $T = T(\bar{y}, \bar{z})$. Since this is a primary model of the hybrid bearing, heat exchange through the pad has not been considered, therefore, being a future step in research.

The term $\partial T / \partial \bar{x} \Big|_0$ refers to the thermal conduction inside the bores. Considering that the bore length is short, one simplifies this term, as follows:

$$\frac{\partial T}{\partial \bar{x}} \Big|_0 \approx \frac{T_{\text{inj}} - T}{l_0} \mathcal{G}(\bar{y}, \bar{z}), \quad (5)$$

where the bore positioning over the pad surface is described by the function $\mathcal{G}(\bar{y}, \bar{z})$, given by

$$\mathcal{G}(\bar{y}, \bar{z}) = \begin{cases} 1 & \text{if } (\bar{y} - y_o)^2 + (\bar{z} - z_o)^2 < \frac{d_0^2}{4}, \\ 0 & \text{if } (\bar{y} - y_o)^2 + (\bar{z} - z_o)^2 \geq \frac{d_0^2}{4}. \end{cases} \quad (6)$$

As boundary condition to solve the energy equation, one supposes a constant temperature distribution at the inlet of the pad. This distribution is resulted from an energy balance applied to the mixing zone (area between pads). The resulting temperature at the inlet of the pad is calculated using the mean temperature of two flows: one passing through the area of the previous pad-rotor gap, and another coming from the supply

(conventional lubrication). Adiabatic temperature boundary condition is also supposed among fluid, bearing pads, rotor and bearing housing. One considers that the lubricant viscosity varies exponentially as function of the temperature.

From the solution of the Energy equation one can determine the temperature distributions of the lubricant along the bearing pad surfaces.

3. Theoretical results

The finite difference method (FDM) was used for solving the energy and modified Reynolds equations. A bi-dimensional uniform grid is created over each pad surface and composed of 62 points in the tangential direction (direction of shaft rotation: \bar{y}) and 101 points in the axial direction of shaft (\bar{z}). Central approximations of the FDM are adopted when discretizing the Reynolds equation, whereas backward approximations of the FDM are adopted when discretizing the energy equation. A Newton–Raphson method was applied to calculate the equilibrium position of the rotor-pads system. Integrating the pressure distributions over the pads, the resulting forces acting between rotor and pads are achieved, and a balance of forces and moments over the rotor and pads can be done, resulting in new equilibrium positions of the rotor-pads system.

Temperature distribution is evaluated after the equilibrium position is determined. Assuming an initial temperature and viscosity distribution, the energy equation (4) is solved. A new distribution oil viscosity over the pads is calculated after the convergence of the temperature distributions over the pads reaches a tolerance of 0.1°C .

A flowchart of the numerical procedure can be seen in Fig. 2 and the bearing characteristics are listed in Table 1.

In order to investigate the influence of different arrangements of bores on the fluid flow and thermal effects of the hybrid bearing, a rotor angular velocity of 50 Hz was set, while the rotor is subjected to an external load of 400 N in the vertical direction (towards pad number 4 – see Fig. 1). Six different arrangements of bores on pads were tested. Basically, two types of bores were used, with diameters of 5 and 2.8 mm. The number of bores in each pad was chosen in order to have a constant total area of bores in all arrangements (5 bores of 5 mm or 15 bores of 2.8 mm). The six arrangements are shown in Fig. 3.

3.1. Rotor centering

When an external load is applied to the rotor, the rotor remains in a position which is not the center of the bearing and can be measured as the eccentricity ratio (ratio between the rotor displacement Y_R and the bearing assembled clearance h_0). Since the hybrid bearing in study works with injected oil through the pads, the performance of such pads (Fig. 3) was investigated by using different injection pressures in pads 2 and 4 (pads in the vertical direction – loading direction). The range of pressures used was 0.1–0.4 MPa, range of which the rotor is centered in the bearing when subjected to an external load of 400 N (Fig. 4). Pressure in pad 4 was always higher than the pressure in pad 2, since the load was applied towards pad 4.

As one can see in Fig. 4, it is possible to replace the rotor into the bearing center by using this kind of lubrication. When one compares the hybrid case to the conventional case (without injection), one can see that better results are obtained by using the hybrid bearing with proper values of injection pressure. However, when a low difference pressure is applied in the pads, one can achieve worse results than those of the conventional case: higher eccentricity ratios (see 0.1 MPa – Fig. 4). This occurs due to the hydrodynamic pressure distribution over the pads. When the injection pressure is not high enough to overcome the hydrodynamic pressures in the bearing gap, an outlet flow of oil may happen from the gap into the bores, therefore, reducing the bearing load capacity. Hence, the presence of bores in areas of low hydrodynamic

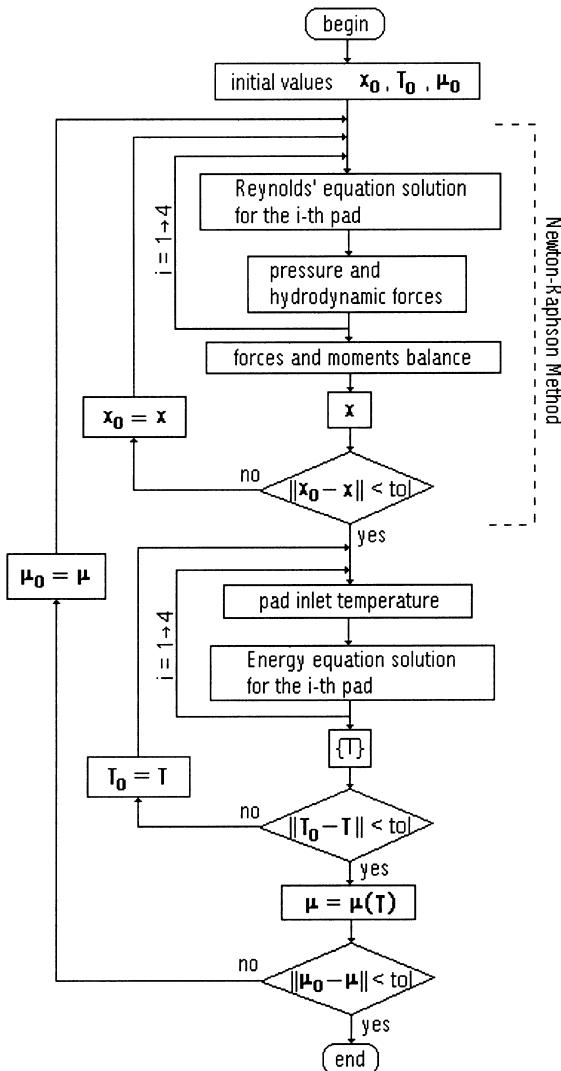


Fig. 2. Flowchart of the numerical procedure.

pressure on the pad's surface is important to ensure an effective injection of oil into the gap, through the bores.

This fact explains the better performances of pads 5-5-5, 7-8 and 2-1-2 in comparison to the other arrangements. If one considers the distribution of bore areas on the pad surface in relation to its center (Table 2), one observes that these are the arrangements with higher percentuals of bores near the pad edges, where the hydrodynamic pressures are lower ($0.3 < d/L_z < 0.5$). Thus, one can replace the rotor to the bearing center by using a difference of injection pressures of only 0.25 MPa with a 5-5-5 pad, whereas pressures of 0.3, 0.32 and 0.4 MPa are necessary when using the 7-8, 21-2 and 15, 2-3 and 5 arrangements. Results presented in Table 2 were obtained summing the areas of the orifices which were positioned in a given distance d to the pad center, and later calculating the percentuals within certain ranges (Fig. 5).

Table 1

Bearing geometry and operational conditions taken into account in the simulations

number of pads	n	4
width/diameter ratio	L_z/D	0.56
pre-load factor	c	0.21
assembled clearance	h_0 (m)	0.0001
rotor radius	R (m)	0.04937
inner pad radius	R_s (m)	0.0495
bore length	l_0 (m)	0.005
rotating speed	Ω (Hz)	50.0
injection temperature	T_{inj} (°C)	35.0
oil supply flow per pad	Q_r (m ³ /s)	0.00004
oil density	ρ (kg/m ³)	840.0
oil thermal conductivity	k_c (W/m K)	0.14
oil specific heat	C_p (J/kg K)	1800.0
oil viscosity (oil type ISO VG 68)	μ (N s/m ²)	$0.06e^{0.045(40-T)}$

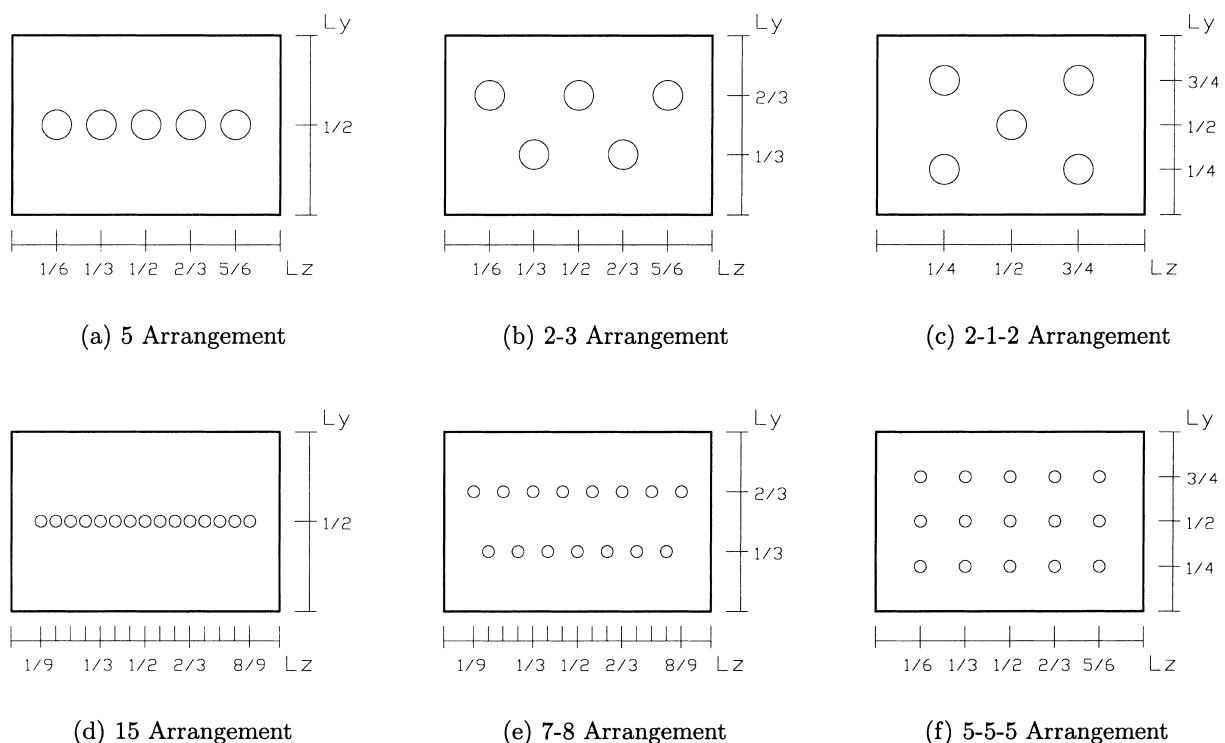


Fig. 3. Arrangements of orifices in the pads.

Fig. 6 shows a comparison between arrangements 5-5-5, 2-3, 5 and conventional, of the pressure distribution in the pad centerline, in both \bar{y} and \bar{z} directions. One can see that one has an increase of pressure in areas where there is not high pressures in the conventional case. This is the cause of a better performance of pad 5-5-5 in centering the rotor, since this arrangement increases the pressure near the edges in both directions.

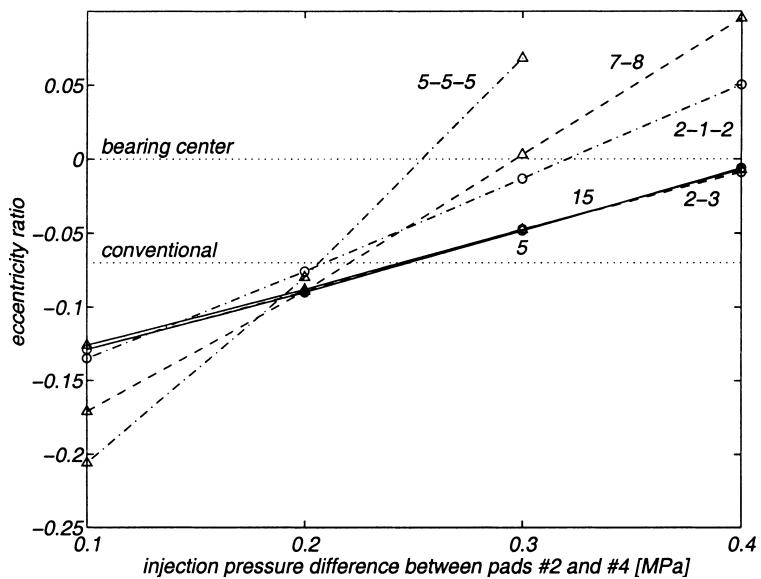


Fig. 4. Rotor eccentricity ratio using different pad arrangements – loading of 400 N.

Table 2
Percentage of orifice areas over the pad surface in relation to the pad center

d/L_z range	Arrangements					
	5	2-3	2-1-2	15	7-8	5-5-5
0–0.1	11.7	0	14.7	21.3	0	4.2
0.1–0.2	39.8	23.4	0	24.2	27.9	14.2
0.2–0.3	9.7	36.5	0	25.9	33.3	30.8
0.3–0.4	38.8	36.9	85.3	26.4	26.8	30.2
0.4–0.5	0	3.2	0	2.2	12.0	20.6

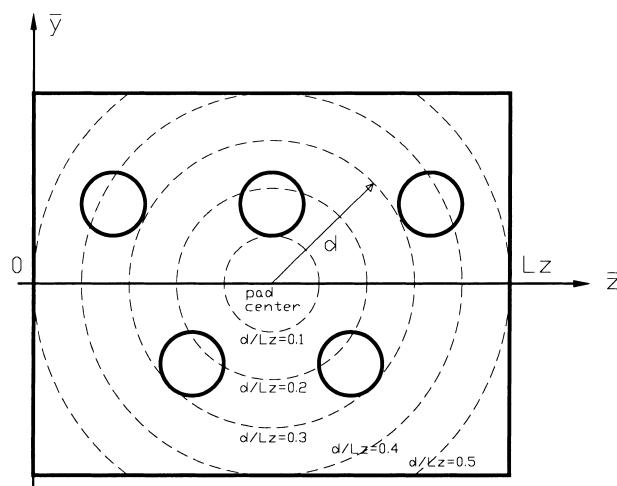


Fig. 5. Evaluation of orifice area distribution in relation to the pad center.

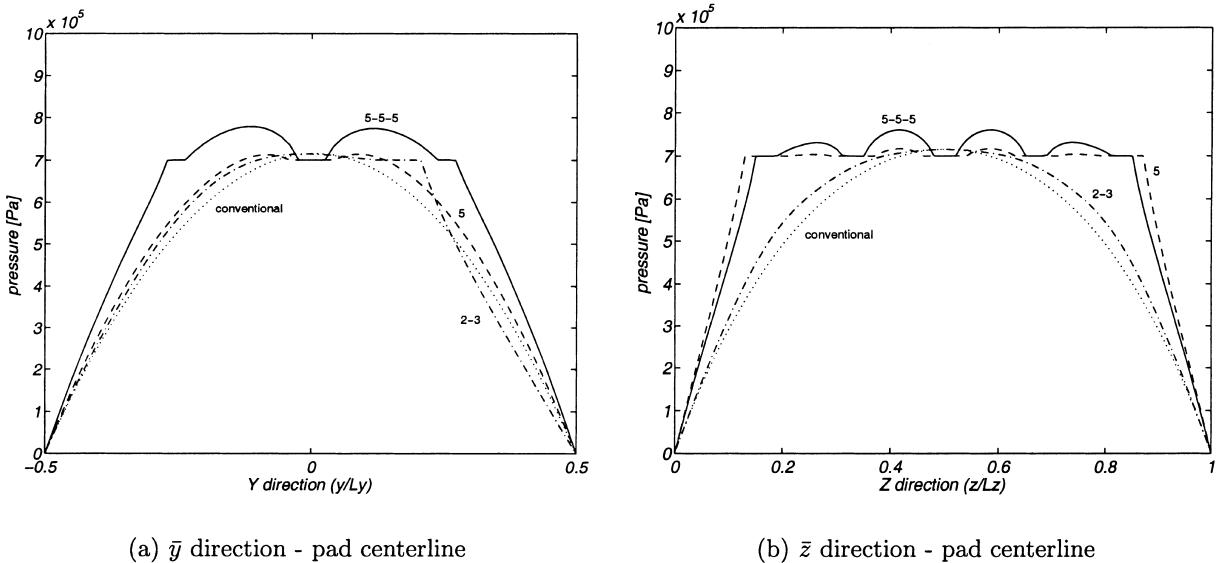


Fig. 6. Oil pressure distribution in the pad centerline for different arrangements – $P_{\text{inj}} = 0.7$ MPa.

3.2. Oil temperature reduction

In order to investigate the efficiency of different types of pads in cooling the oil flow in the bearing gap, by injecting cooled oil through the bores, a difference of injection pressures between pads 2 and 4 was used, within the range of 0.1 and 0.4 MPa. The average oil temperature was calculated over pads 2 and 4, and the results are shown in Figs. 7 and 8.

Fig. 7 shows that the average oil temperature over the pads are very similar to that of the conventional case (without injection), when the injection pressures are low (between 0 and 0.3 MPa in each pad). In this case, when the injection pressure is low, an outlet flow of oil from the gap into the bores occurs, and there is no cooling effect. However, as the injection pressure is increased, the hydrodynamic pressures are overcome and an inlet flow of cooled oil into the gap occurs, and the average temperatures decrease.

Due to the vertical loading towards pad 4, the temperatures over this pad are higher than those over pad 2. As soon as the rotor is replaced to the bearing center (due to the increase of injection pressure difference between pads), the temperatures in pad 4 reduce and in pad 2 increase (Figs. 7 and 8). In the case of injection pressure difference of 0.4 MPa (Fig. 8(b)), when there is an eccentricity ratio in the opposite direction of the load (Fig. 4), the temperatures on pad 4 are lower than those on pad 2, which is near to the temperatures reached in the conventional lubrication case.

Comparing the performance of the different arrangements of bores in pads, one can see in Figs. 7 and 8 that the lower average oil temperatures are obtained with the 7-8 pad, in most of the cases. Low temperatures are also reached with the 15 arrangement, but the 7-8 arrangement is more sensitive to the increase of injection pressure (temperatures are reduced with lower injection pressures). This sensibility to the injection pressure is caused by the presence of bores in areas of low hydrodynamic pressures (near the pad edges, Table 2). In these areas, lower pressures can overcome the hydrodynamic pressure and an higher volume of cooled oil can enter the gap.

However, the 15 arrangement does not present a good cooling effect, although this type of pad has a good distribution of bores near the pad edges (Table 2). The fact is that, as soon as there is an inlet flow of cooled oil through all the bores in the pad, what makes difference in the average oil temperature is the distribution of bores in the \bar{z} direction. The velocity components of the oil flow over the pad in \bar{y} direction

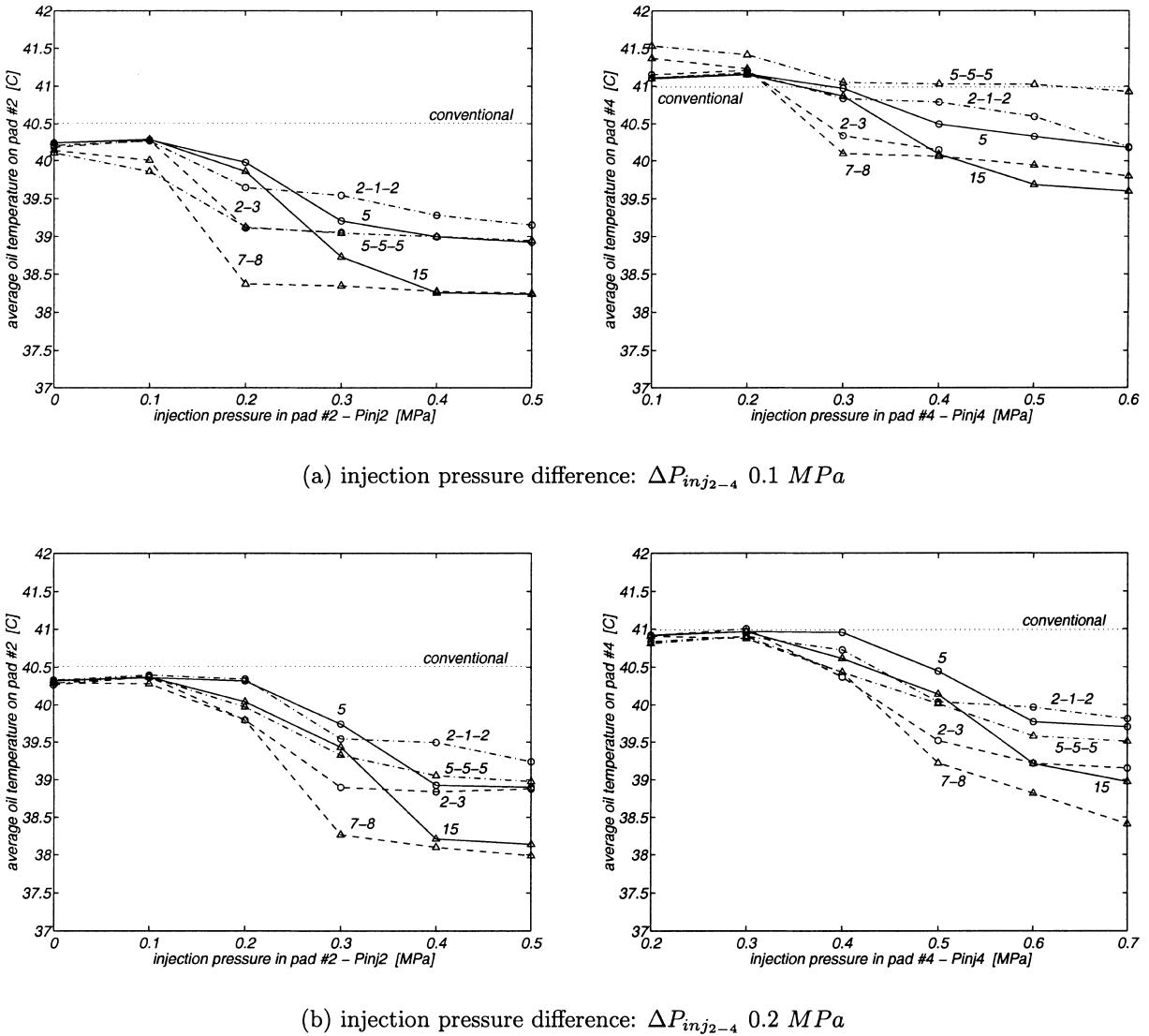


Fig. 7. Average oil temperature over pads 2 and 4 – variation of injection pressure.

are larger than those in \bar{z} direction. Thus, if a pad has its bores concentrated in \bar{z} direction (case of aligned bores – 5-5-5 and 2-1-2 arrangements), the cooled oil stream will be restricted to the area forward the bores, not having a wide cooling of the oil flow in the gap (Fig. 9). Therefore, those pads with a better distribution of bores along the \bar{z} direction and near the pad edges are the those with a best performance in cooling the oil flow in the gap, using lower injection pressures.

4. Concluding remarks

A thermohydrodynamic analysis was led in TPJBs with hybrid lubrication by multiple orifices. Different types of orifice arrangements in pads were tested, and the main results are listed below:

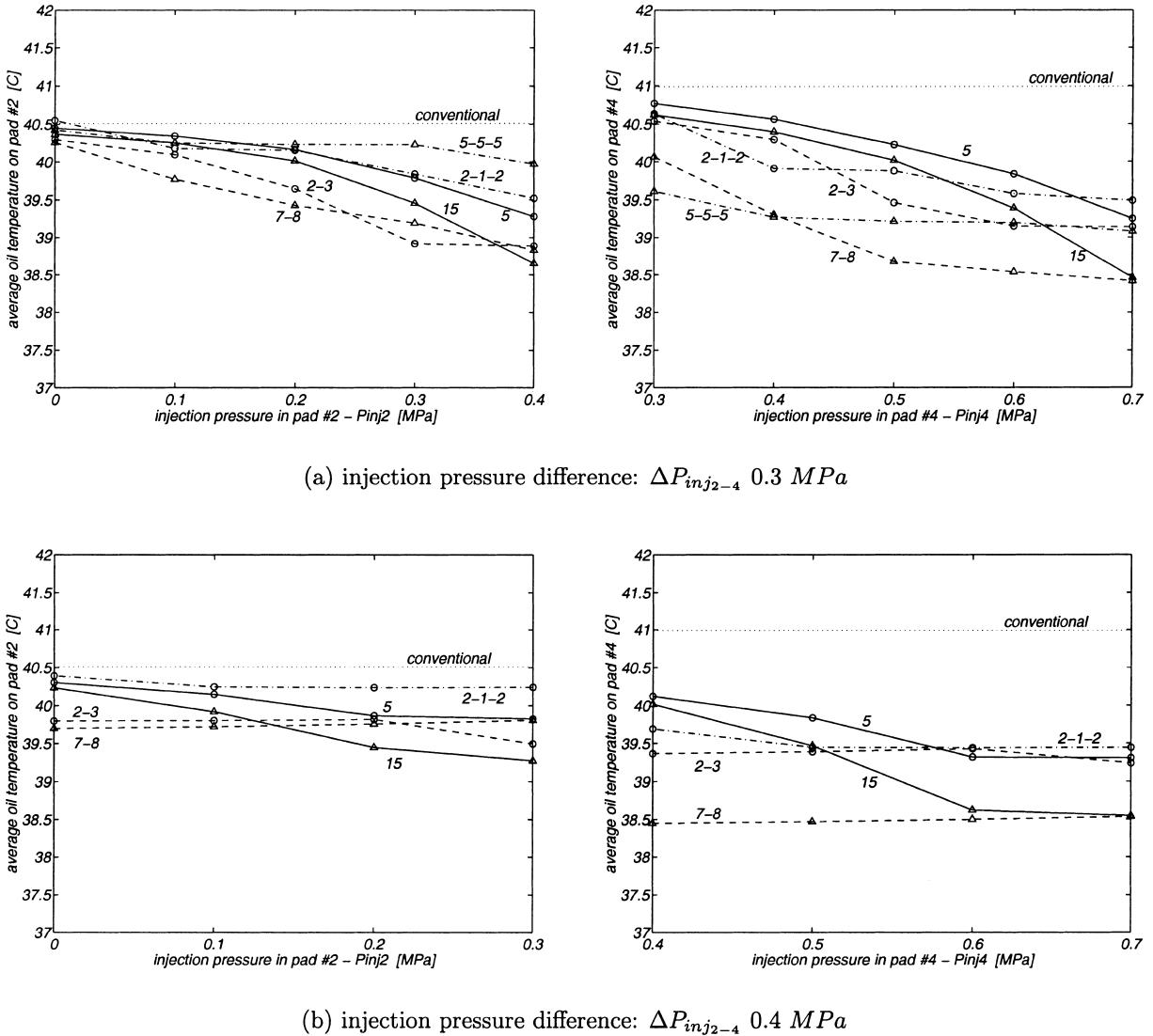


Fig. 8. Average oil temperature over pads 2 and 4 – variation of injection pressure.

(1) Numerical results show that it is possible to control the rotor equilibrium position by using this kind of hybrid lubrication.

(2) A decrease of bearing load capacity occurs when the injection pressure is lower than the hydrodynamic pressures in the gap. The oil flows from the gap into the bores and similar temperatures to the conventional case are obtained, since there is not an inlet flow of cooled oil into the gap.

(3) The presence of bores in areas of low hydrodynamic pressure on the pad's surface (near pad edges) is important to ensure the inlet flow of oil into the gap. The best results in controlling the rotor with lower injection pressures were achieved with pads with higher percentuals of bores near the pad edges (5-5-5, 7-8 and 2-1-2 arrangements).

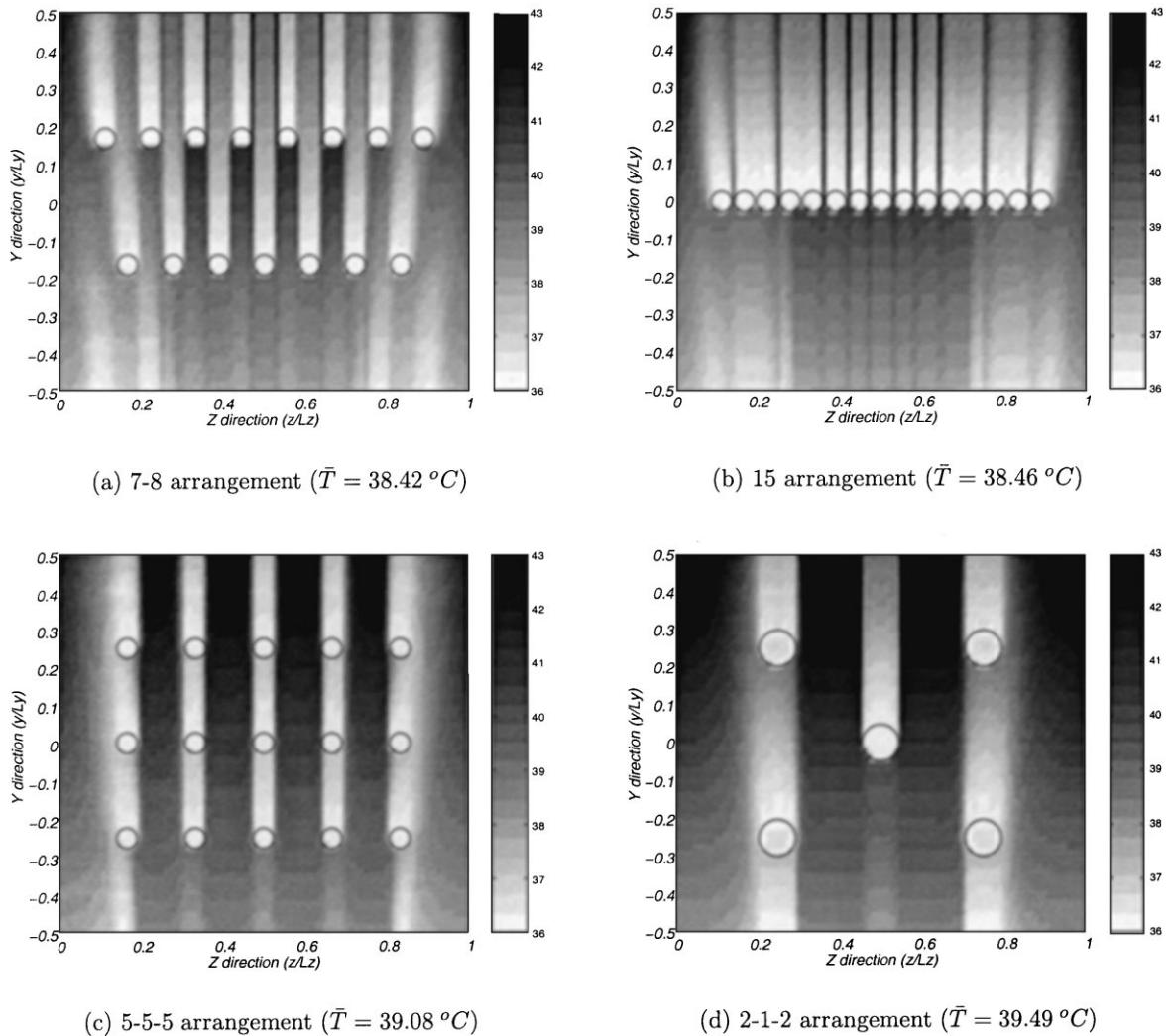


Fig. 9. Oil temperature distribution over pad 4 – $P_{\text{inj}} = 0.7 \text{ MPa}$, $\Delta P_{\text{inj}_{2-4}} = 0.3 \text{ MPa}$.

(4) As soon as there is inlet flow of cooled oil through all the bores in the pad, the distribution of bores along the \bar{z} direction is important to the decrease of oil temperatures over the pad surface. The best results were obtained with 7-8 and 15 arrangements (pads with better orifice distribution along \bar{z} direction).

(5) The presence of bores near the pad edges increase the sensibility of the temperature to the injection pressure. Lower injection pressures are needed to reduce the average oil temperatures over the pads, when there is a high percentual of bore area near the pad edges.

It is important to point out that, the bearing in study is not hydrostatic, since there are no pockets in the pads, and its load capacity comes mainly from the hydrodynamic forces. The injection of oil through orifices has the purpose of controlling the rotor movements, and cooling effects are a secondary advantage of the system.

Acknowledgements

The Brazilian research foundation Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP) is gratefully acknowledged by the support given to this project.

References

Brockwell, K., Dmochowski, W., 1992. Thermal effects in the tilting pad journal bearing. *J. Phys. Part D: Appl. Phys.* 25, 384–392.

Dowson, D., 1962. A generalized reynolds equation for fluid film lubrication. *Int. J. Mech. Sci.* 4, 159–170.

Ettles, C.M.McC., 1980. The analysis and performance of pivoted pad journal bearings considering thermal and elastic effects. *ASME J. Lubricat. Technol.* 102, 182–192.

Fillon, M., Khonsari, M., 1996. Thermohydrodynamic design charts for tilting-pad journal bearings. *ASME J. Tribol.* 118, 232–238.

Flack, R.D., Zuck, C.J., 1988. Experiments on the stability of two flexible rotor in tilting-pad. *J. Bear. Tribol. Trans.* 31 (2), 251–257.

Heshmat, H., Pinkus, O., 1986. Mixing inlet temperatures in hydrodynamic bearings. *ASME J. Tribol.* 108, 231–248.

Jones, G.J., Martin, F.A., 1979. Geometry effects in tilting pad journal bearings. *ASLE Trans.* 22 (3), 227–244.

Santos, I.F., 1993. Active Tilting-Pad Journal Bearings: Theory and Experiment, vol. 11. VDI Verlag, Dusseldorf, p. 189 (in German).

Santos, I.F., Nicoletti, R., 1996. Self-excited vibrations in active hydrodynamic bearings. *RBCM J. Braz. Soc. Mech. Sci.* 3, 263–272.

Santos, I.F., Nicoletti, R., 1998. THD analysis in tilting-pad journal bearings with hybrid lubrication. *ASME J. Tribol.* 121 (4), 892–900.

Santos, I.F., Russo, F.H., 1998. Tilting-pad journal bearings with electronic radial injection. *ASME J. Tribol.* 120 (3), 583–594.

Santos, I.F., Scalabrin, A., 2000. Control design for active lubrication with theoretical and experimental examples. *ASME/IGTI 2000 Gas Turbines Conference and Exposition*, Munich/Germany, May 1–5, 2000, in press.

Tanigushi, S., Makino, T., Takeshita, K., Ichimura, T., 1990. A thermohydrodynamic analysis of large tilting-pad journal bearing in laminar and turbulent flow regimes with mixing. *ASME J. Tribol.* 112, 542–548.

Ulbrich, H., Althaus, J., 1989. Actuator design for rotor control. *Twelfth Biennal ASME Conf. Vibr. Noise*, Montreal/Canada, pp. 17–22.